289. Proposed by G. W. DROKE, Professor of Mathematics, University of Arkansas.

Find the curve such that the rectangle under the perpendiculars from two fixed points on the normals be constant.

Solution by S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, N. Y.

This is problem 27, page 61, Murray's *Differential Equations.*

Take the straight line through the fixed points as the x axis, and the middle point as the origin. The coordinates of the points are then $(a, 0)$ and $(-a, 0)$. Using p for $\frac{dy}{dx}$, the equation of a normal at (x', y') is

$$y-y'=-\frac{1}{p}(x-x').$$

Its distance from $(a, 0)$ is

$$\frac{\frac{a}{p}-\left(y'+\frac{x'}{p}\right)}{\sqrt{1+\frac{1}{p^2}}},$$

and its distance from $(-a, 0)$ is

$$\frac{\frac{-a}{p}-\left(y'+\frac{x'}{p}\right)}{\sqrt{1+\frac{1}{p^2}}}.$$

The constant product, dropping primes, is

$$\frac{y^2+\frac{2xy}{p}+\frac{x^2}{p^2}-\frac{a^2}{p^2}}{1+\frac{1}{p^2}}=k^2,$$

or $p^2y^2+2xyp+x^2-a^2=k^2p^2+k^2$, or $(y^2-k^2)p^2+2xyp+x^2-a^2-k^2=0$;

whence the p discriminant is

$$4x^2y^2-4(y^2-k^2)(x^2-a^2-k^2).$$

Hence, $$4x^2y^2-4x^2y^2+4(k^2+a^2)y^2-k^2(x^2-a^2-k^2)=0,$$

$$\text{or, } \frac{y^2}{k^2}+\frac{x^2}{k^2+a^2}=1.$$

This is the singular solution of the differential equation. It represents a system of confocal conics having the fixed points as foci.

Also solved by V. M. Spunar, G. B. M. Zerr, and J. Scheffer.

Solution by PROFESSOR F. L. GRIFFIN, Williams College.

However the ellipse is placed relative to the co-ordinate axes, its area is

$$S=\int_{x_1}^{x_2}(y''-y')\,dx,$$

where x_1 and x_2 $[x_1<x_2]$ are the extreme abscissas taken in the curve, and y' and y'' $[y'\leqq y'']$ are the two ordinates corresponding to any one value of x. Solved for y, the given equation becomes

$$by=-(hx+f)\pm\sqrt{[f^2-bc+2(hf-bg)x+(h^2-ab)x^2]}.$$

Now let

$$f^2-bc=b^2A,\quad fh-bg=b^2B,\quad ab-h^2=b^2C,$$

where for an ellipse $C>0$. Then the difference of the two ordinates becomes

$$y''-y'=2\sqrt{[A+2Bx-Cx^2]}.$$

Hence, integrating,

$$S=2\left[\frac{Cx-B}{2C}\sqrt{[A+2Bx-Cx^2]}+\frac{B^2+AC}{2C^{\frac{3}{2}}}\sin^{-1}\frac{Cx-B}{\sqrt{[B^2+AC]}}\right]_{x_1}^{x_2}.$$

Now the extreme abscissas make $y'=y''$, or $A+2Bx-Cx^2=0$; whence

$$Cx_2=B+\sqrt{[B^2+AC]}\ \text{ and }\ Cx_1=B-\sqrt{[B^2+AC]}.$$

Substituting these values,

$$S=\frac{B^2+AC}{C^{\frac{3}{2}}}[\sin^{-1}1-\sin^{-1}(-1)]=\frac{\pi}{b}\,\frac{[(fh-bg)^2+(ab-h^2)(f^2-bc)]}{(ab-h^2)^{\frac{3}{2}}},$$

which immediately reduces to the formula proposed.

Also solved by G. B. M. Zerr and J. Scheffer.

MECHANICS.

240. Proposed by S. A. COREY, Hiteman, Iowa.

A perfectly flexible wire rope weighing one pound per foot is suspended from the tops of two vertical supports 300 feet apart, one support being 30 feet higher than the other. One end of the rope is fastened to the top of the higher support, while 600 feet of the rope hangs vertically from the top of the lower support. Assuming that the rope is free to slide over the top of the lower support without friction, find the lowest point of